Open-ended Tasks as Stimuli for Learning Mathematics

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[1] This paper is dedicated to the memory of Dianne Bourke, who passed away in February 1995.

Australian Catholic University, Christ CampusConventional teaching, particularly at upper primary and secondary levels, often consists of teacher demonstration of one or more exercises, with explanations and demonstrations linked to examples. Students predominantly work on drill and practice exercises. If the feedback is negative, students do more practice; if the feedback is positive then the class moves on to new exercises. Students are likely to see mathematics as a collection of rules and exercises. Christiansen and Walther (1986) suggested, by contrast, that teaching should be based on carefully chosen activities of the constructive, exploratory and problem-solving types which allow the teacher to build on the learner's personal activity. They contrasted the notion of task and activity. The task is the object of the students' activity, and educational activity is when pupils work as a result of some plan. Learning activity is when learning as intended takes place. They give an example of a task:

How much does it cost to keep a dog for a year?

The teacher might hope that pupils construct a plan, organise the work, collect data, systematise, transform, reorganise, and evaluate. The intended learning is the development of processes. The teacher's role is to differentiate between learning needs according to required levels of support. The hope is that students' actions develop into cognitive strategies and over time become schemata.

Christiansen and Walther (1986) argued that non-routine open-ended tasks provide optimal conditions for cognitive development in which new knowledge is constructed and items of earlier acquired knowledge are recognised and evaluated. In a conventional milieu, good teaching is equated with providing clear explanations followed by effective drill and practice. With teaching based on the learners' activity on well chosen tasks, the teacher must respond differently. There is a need to avoid working through introductory examples, but it is important to support appropriate on-going activity through the lesson, and to sum up and reflect with the whole class (Christiansen & Walther, 1986).

It is also possible to devise open-ended tasks which address dimension 2 learning and which focus on particular components of mathematics. Krainer (1992) referred to the poles of the dilemma where on one hand we have mathematics as a complex and developed science, and on the other hand the need to acknowledge the spontaniety and creativity which students bring to their classes. Krainer (1992) described "powerful tasks" which are more than problems, but may deal with describing or discussing a situation. Powerful tasks are open-ended allowing the pupils to pose and discuss new questions. Among other features, the tasks stimulate a high level of acting and a high level of reflecting. One example of such tasks, taken from the Shell Centre (1985) is where students are asked to describe which of a variety of line graphs represent the way a student might hoist a flag on the school's flagpole.

Nohda (1986) recognised the connection between the task set and the type of mathematical thinking in which the learners engage. He described an "open-approach" method for teaching which combines both open-ended problems and problems for which there are multiple solution paths. He suggested that the open-approach teaching method fosters not only creative thinking in students but also mathematical activity at the same time. He presented an example where a diagram (the O and X represent different coloured marbles) is presented as follows:

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O        OX       OXO      OXOX
         XX       XXO      XXOX
                  OOO      OOOX
                           XXXX
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The students were asked to construct their own problems. Nohda reported that some of the problems the students constructed were:

How many marbles are there when each side of the square has 10 marbles? In such a square how many X's will there be?

Nohda (1986) explained that the task is open in three ways. First, there is the openness in the students' activity. The main point here is that the questions are created by the students. This greatly contributes to the motivation to solve the problem. Second, there is openness in the mathematical content. Not only is the same mathematical potential present here as in text book tasks, there is also the possibility of generalisation and diversification of the problem. Third, there is the openness of interaction between the students and the mathematical content. In this Nohda contrasted coventional teaching where the teacher plans the lesson and approach beforehand with this mode where the students' problems and solutions are considered by the teacher and then used by the teacher as the basis of further tasks. He also noted that this approach caters for a range of abilities within the class.

In Nohda's tasks the openness arises primarily because of the variety of solution paths to a task and in the creativity necessary to invent their own questions. A similar approach was taken by Pehkonen (1992) who created "problem fields" out of which mathematically rich open-ended explorations are generated. A problem field consists of clear but rich problems from which many different explorations can arise. For example, the task: *With 12 matchsticks make a square with an area of 9 area units.* can be extended to include other investigations such as:

How many different polygons of 5 area units can you make with 12 matchsticks? Can there be more than ten different solutions?

Such problem fields can stimulate the conditions for learning listed above mainly because after the initial tasks, the students can influence the direction and goals of the investigations.

One dilemma with each of the approaches described is that teachers can be tempted to use such tasks as additions to the program, rather than as core activities. One approach which can help introduce open-ended tasks to mainstream teaching was described by Sullivan and Clarke (1988). They used the term "good questions" to describe a style of open-ended tasks which are also content specific. One example of a good question is:

My vegetable garden is shaped like a rectangle. The perimeter of the garden is 30 metres. What might be the area of my garden?

This question is different from conventional perimeter and area questions in two major ways. First, it requires a higher level of thinking and engagement than do conventional questions. Traditionally mathematical questions have required students to repeat a procedure or recall an algorithm. The sample question engages students in constructive thinking by requiring them to contrast the related concepts of perimeter and area and to think about relationships for themselves. Another advantage of the question over conventional items is that the need for thinking by individual students is made clear to the student. The students cannot rely on remembering a rule or simply manipulating formulas, they must think about the concepts, their meaning and the links between them. Further, Cobb (1986), Doyle (1986) and Desforges and Cockburn (1987) all noted the tendency for students to respond adversely to higher order tasks by seeking to have the demand of tasks made more explicit. Good questions have the potential to overcome this adverse reaction by stimulating higher level thinking within a specific framework.

Second, the question has more than one possible appropriate answer. Some students might give just one response, others might produce many appropriate answers, and there may be some who will make general statements. The openness of good questions offers significant benefits to classroom teachers because of their potential for students at different stages of development to respond at their own level (Sullivan, Clarke & Wallbridge, 1990).

Among other advantages for classroom teachers is that good questions are suited to group learning, and they focus the attention of students onto aspects of mathematics such as generalising, and identifying patterns and relationships. Good questions allow students to be creative, to work with others in responding to set tasks, and to recognise that many problems have multiple solutions. An important feature of the questions is that learning occurs as an outcome of the students' explorations and thinking, not as a result of listening to the teacher.

Other examples of open-ended tasks which can be called good questions are:

A number has been rounded off to 5.8. What might the number be?

Draw some triangles with an area of 6 sq. cm.

Find two objects with the same mass but different volume.

Describe a box with a surface area of 94 sq. cm.

A further feature of these open-ended tasks is that they are focused explicitly on which could be termed the syllabus content of mathematics. Even though mathematical knowledge has been described as solely "mathematical activity" (Wheatley, 1991, p.11), and it has been argued that social relevance be the sole criteria for the focus of mathematics curricula (Ernest, 1991), it must also be recognised that mathematical concepts are part of the culture in which we live, and so form part of the inculturation of our young into our society. Further, as Putnam, Lampert and Peterson (1989) explained in a review of alternative perspectives on knowing mathematics, the feature which distinguishes expert problem solvers is the rich store of organised accessible knowledge and ways of representing problems. An appreciation of underlying mathematical structure is essential for this. We must consider how can we best create the conditions for our young (and not so young) to learn key mathematical concepts which are necessary to make people truly free in their society. This will include explicit treatment, at times, of these key concepts within mathematics classes.

In summary, there is increasing use of open-ended tasks as a way of encouraging students to become learners of mathematics by doing and even creating mathematics for themselves. Open-ended questions have the potential to allow students to respond to questions in their own way, they offer teachers of heterogeneous groups a method for catering for the diverse ranges of interests and experience in the class, and they also allow the focus of the explorations to be mathematics. The next step is to examine ways in which such questions can be used in mathematics classes and to consider the responses of students to working on such tasks. This is the report of one such investigation.

A classroom investigation of classroom activity based in the use of open-ended tasks

This is the report of a teaching program which was based as far as possible on the use of open-ended questions. The goal was to describe what happened when the primary activity in a mathematics class was the students' activity on open-ended tasks rather than the teacher's transmission of information. The teacher was a graduate student with 10 years teaching experience who had expressed an interest in the use of open-ended questions. The class was at grade 6 (age 11, 12) level in an outer suburban primary school. While the community could be described as lower socio-economic status most families would have at least one employed adult and nearly all would speak English at home.

Since changes in learning, attitudes, or mathematical understanding generally develop over long periods examination of brief programs is difficult. In this case the data collection period was brief because of the breadth of data collected, but also because the teaching style was sufficiently different from common practice at the school that to extend the investigation may have been intrusive. It was hoped to learn about the effect on the teacher, the pupils, and their learning from a program based solely on open-ended questions.

One major concern was that the students would be unfamiliar with the style of the questions. To overcome this, the teacher was asked to use a range of good questions in each of the preceeding topics. The observation period was 10 weeks into the school years. Prior to the program there was also a session during which the purpose of the program and the style of teaching were discussed. Examples of good questions from mathematics and other disciplines were given and there was discussion of the types of responses possible. The way these question differ from conventional exercises was also discussed with the students.

The data on which the findings were based included structured and unstructured observations of the teacher, the students' responses to tasks posed in class, summary of the responses of students to closed and open-ended questions, diaries completed by the students after each lesson, responses to two attitude instruments, information from the observations of the students, and transcripts of interviews with students. Due to space limitations, only the responses to the written tests are presented here.

A detailed program was given to the teacher. The information included a focus for each lesson, a selection of good questions, and some suggested conventional exercises. As an example of the information provided for the teacher, the following was provided for lesson 3:

Purpose	Key activities	Other activities
Lesson 3. *Using metres, cm and mm., and introducing perimeter*	Draw a rectangle which is 12 cm around (*on squared paper*). Find something which is 10 cm long. Find some things which are twice as long as they are high. *Give the students a piece of string. Ask* The string is the distance around some objects. What might some of the objects be?	Measure width and height of specific objects using rulers in m and cm Estimate, then measure, a collection of lines, some straight, some curved Arrange a collection of containers (e.g. bottles, jugs) by perimeter

Individual worksheets with space for written responses for each of the good questions were prepared prior to the program and presented in the format of a workbook.

There were 45 explanation events overall; 33 were coded a relational, 10 as instrumental, and 2 as other. Given that an explanation event may be a brief as a single sentence, this represents an average of only five explanations per observed lesson.

There was a range of additional data collected to provide an impression of the content and style of the lessons. These included:

- an unstructured summary of the teacher's actions by the observer for each lesson;
- a diary completed by the teacher daily to record her reflections; and
- recording of the lesson review.

Overall the data suggest that the implementation of the program was compatible with the intention which was that the questions and tasks would be open-ended and require creative input from the students, and that there would be few teacher directed explanations.

Written test responses

One of the measures of the outcomes of the program was from the students' responses to two written tests which they completed before the program (pre-test), after the program (post-test), and again three months after the teaching program (delayed post-test).

The first test consisted of 12 closed items. Six multiple-choice items were taken from the PAT test series (Australian Council of Educational Research, 1984) which have normed data available, and a further six items were direct questions which required the students to demonstrate aspects of the content of the program. Even though there were few closed items presented as part of the teaching, it was anticipated that students would learn aspects of calculating length, perimeter and area and be able to solve word problems associated with those concepts from their explorations of the open-ended tasks. The first test also sought to provide opportunities for the students to demonstrate their interpretation of the concepts. For example, one item was:

On this page, draw a line 50cm long.

This item required the students to realise that the line could not be straight, since their pages were about 30cm long, and also to measure the length of the line they drew.

The second test consisted of six open-ended items. These items also required an appreciation of the concepts in the program, and provided the students with the opportunity to show whether they could apply the concepts to various practical situations, and even to recognise the possibility of a range of answers. The items were similar in style to the tasks in the program. An example of one such item is:

A shape made from a sheet of metal has an area of 60 square cm. What might the shape look like?

A comparison of the means of the three administrations of test 1 is presented in Table 1. The results presented are for the 30 students who completed all three tests.

Table 1: Comparison of three administrations of test 1 (n=30)

	Mean	Std. Dev.
Pre-test	5.1	2.17
Post-test	6.5	2.33
Delayed post-test	7.5	2.45

An analysis of variance, performed using the Statview analysis package (Feldman & Gagnon,1986) , indicates that the means are significantly different (F=2.069, df=2,29, p<.001). The Fisher PLSD post hoc procedure was applied to each of the pairs, and this indicated that the mean of the post-test is significantly different from the mean of the pre-test (p<.05) and that the mean of the delayed post-test is significantly different from the mean of the post-test (p<.05). This suggests that the students overall were better able to complete the items after the program than they were before the program. There was some maturation which resulted in further improvement in the test scores. The class teacher did not specifically address these concepts in the intervening mathematics classes.

The purpose of test 2 was to determine whether the students were able and willing to give one or more appropriate responses to the open-ended items. To allow comparison between the administrations of the test, the responses were scored as 0 for an incorrect response, and 1 for one or more appropriate responses. A comparison of the means of the three administrations of test 2 are presented in Table 2. The results presented are for the 27 students who completed all three tests.

Table 2: Comparison of three administrations of test 2 (n=27)

	Mean	Std. Dev.
Pre-test	2.6	1.41
Post-test	3.9	0.93
Delayed post-test	3.6	1.39

An analysis of the variance indicates that the means are significantly different (F=1.776, df=2,26, p<.04). The Fisher PLSD post hoc procedure was applied to each of the pairs, and this indicated that the mean of the pre-test is significantly different from both the mean of the post-test (p < .05) and the mean of the delayed post-test (p < .05). This suggests that the students overall were better able to complete the items after the program than they were before the program. The overall results for the delayed post-test were similar to the post-test.

While such results give some indication of trends, it is more helpful to examine the results of particular items. To allow comparison with the program, the items which focus on perimeter are presented in the following tables. Table 3 presents two items from test 1 which addressed the concept of perimeter.

Table 3: Perimeter items from test 1 (percentage correct)

Item	Pre-test n=31	Post-test n=32	D. post-test n=32
What is the perimeter of this shape? (A diagram of a rectangle 10cm X 4cm was drawn, but the lengths of the sides were not shown)	16%	72%	72%
Draw a rectangle with a perimeter of 8cm.	47%	75%	68%

Again more students were able to respond correctly after the program and the proportion of students responding correctly was similar 3 months after the program. The first of these questions seems to have been more difficult initially, but there was a similar proportion of students responding correctly after the program.

The following scoring was used for the open-ended items.

0 meant no appropriate responses were given

1 meant only one appropriate response was given

2 meant two or three appropriate responses were given

3 meant all or many appropriate responses were given

4 meant a general response was presented.

Table 4 given a breakdown of the response types for one item given by the students. The figures presented are the number of students who gave that type of response.

Table 4: The number of students giving particular response categories for the open-ended items

Item	Response code	Pre-test n=30	Post-test n=31	D. post-test n=32
	0	8	0	5
A shape has a perimeter of 16cm. What might the shape look like?	1	21	3	20
	2	1	22	4
	3	0	6	3
	4	0	0	0

There is a noticeable difference between the pre- and the post-tests. Al students gave at least one correct response, 2/3 gave more than one response, and 1/5 gave all or many possible responses. This seems to be a positive outcome of the program and reinforces the assertion that students can respond to the tasks at a variety of levels. By the delayed post-test, the profile of responses is more like the pre-test. While some continued to give more than one response, most gave just one response and 5 were not able to respond.

The results suggest that the students may have learnt the basic concepts as an outcome of their activity on the open-ended tasks, and that this was retained and even extended over time. On the other hand, the tendency to give multiple responses diminished markedly on the delayed post-test suggesting that it was not the normal mode for their responses.

One interesting aspect of the responses to the test items is what seems to be anomoly in their order of difficulty. It is useful to examine three of these items.

P1. What is the perimeter of this shape? (A diagram of a rectangle 10cm X 4cm was drawn, but the lengths of the sides were not shown)

P2. Draw a rectangle with a perimeter of 8cm.

P3. A shape has a perimeter of 16cm. What might the shape look like?

It was assumed that these items would increase in difficulty for the students in the order presented. This was not the case. Table 5 compares the responses of the students on items P1 and P2.

Table 5: Responses of students to both P1 and P2.

	No. of students		
	Pre-test	Post-test	D.post-test
P1 and P2 incorrect	10	4	5
P1 correct and P2 incorrect	4	5	4
P1 incorrect and P2 correct	6	4	5
Both P1 and P2 correct	10	19	18

There is no indication that the students responded more easily to one question or the other. It seems that questions P1 and P2 were about the same level of difficulty for the class. Note though that about one third of the class on each test were able to respond to only one of the two questions, even though they seem to be testing very similar concepts.

The surprise is when we compare these responses with those to P3. Table 6 compares the responses with both of these questions and with P3. Note that while P1 and P2 are scored as correct (1) and incorrect (0), the full range of responses described above are used for P3.

Table 6: No. of students responding to items on perimeter on both tests.

Pre-test	P1			P2		
	Responses	0	1	Responses	0	1

	0	5	3
P3	1	8	13
	2	1	0

	0	5	3
P3	1	10	11
	2	1	0

Post-test

P1

	Responses	0	1
	0	0	0
P3	1	2	0
	2	5	17
	3	1	5

P2

	Responses	0	1
	0	0	0
P3	1	2	0
	2	4	18
	3	1	5

Delayed post-test

P1

	Responses	0	1
	0	1	4
P3	1	7	13
	2	1	3
	3	0	3

P2

	Responses	0	1
	0	3	2
P3	1	7	13
	2	0	4
	3	0	3

The overall impression is that P3 is the easiest of these items. This is contrary to prior expectations.

The numbers in the cells are too small to permit statistical analysis, but the trends are generally clear. On the pre-test, noticeably more students could respond to P3 than to P1 or P2. Nine and 11 students respectively could answer P3, but did not respond correctly to P1 and P2. The post-test results were even more marked. While 8 and 7 students respectively did not answer P1 and P2, all could give one correct response to P3. There were even students who gave multiple responses to P3, but did not give a correct response to P1 or P2. The delayed post-test reinforces the impression that more students can respond to P3 than P1 or P2.

This is a startling result. An inspection of the three items suggests that they all presume some appreciation of the meaning of the term and the concept of perimeter. It is unfortunate that the items use different numbers, but this is unlikely to be the cause of the differences. This grade 6 class was competent at most aspects of operations of such small numbers. It is hard to see how P1 and P2 measure different skills, but also it seems that whatever is needed to respond to P1 and P2 is also needed to respond to P3, and that some further analytical skills would be needed besides.

The result is certainly contrary to the wisdom of virtually all text book writers. Commonly we see conventional closed questions early in the chapters with maybe one or two open-ended tasks as extension. Many teachers also see open-ended tasks as extension. While the issue requires further investigation, the suggestion is that such open-ended tasks may be useful for initial explorations of concepts and ideas. This suggestion is consistent with the findings of Owen and Sweller (1985) that less specifically stated goals allow more efficient learning since more processing capacity is available to work on tasks and to learn from such exploration.

Summary

This study was based on a belief that open-ended tasks are one way of encouraging students to become learners of mathematics by doing and even creating mathematics for themselves. It aimed to examine the effect of a class program based mainly on open-ended content specific tasks.

The program was planned with the teacher to ensure that the questions were suitable for the class. It seemed that the program was delivered in a way which was compatible with the intentions of the study. Most questions asked were open-ended, and there were few teacher explanations. The students were engaged in personal constructive mathematical activity and there were no management or organisational programs created by the program. Observation of individual students and interviews confirmed these impressions and indicated that teaching based on open-ended questions is suitable both for students who are confident at mathematics and for those who lack confidence.

Students showed significant improvement on a test of closed items based on the content of the program and the improvement was maintained after the program. Students also showed an overall improvement on the open-ended test questions although the tendency to give multiple correct responses was not maintained after the program. A most interesting observation was that it seemed that the open-ended questions were easier for the students both before and after the program than closed questions which were otherwise apparently comparable.

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We acknowledge the assistance of the Australian Catholic University Research Grants for support to enable the project to be conducted.